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13. ABSTRACT (Maximum 200 words) Research was concerned with applying the Neutral Action method in order to derive conservation laws for viscoelastic bodies. In contrast with previous attempts, emphasis was put in constructing conservation laws in the space-time domain and the viscoelastic models considered are therefore given in terms of rate equations. The final report summarizes the results of this effort.  92-28001  1021			
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Conservation Laws for  
Elastic Systems with Dissipation

Final report

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and

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September 30, 1992

U.S. ARMY RESEARCH OFFICE

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## A. STATEMENT OF THE PROBLEM STUDIED

### 1. Motivation

The importance of conservation laws is widely recognized in many fields of physics and mechanics. Conserved quantities such as mass, energy, momentum and angular momentum are fundamental in classical and modern physics. They provide insights into the behavior of various systems, ranging from dynamic of rigid bodies to continuum physics. In the theory of elasticity, conservation laws represented as path-independent integrals (such as  $J$ ,  $L$  and  $M$ ) have captured the interests of many researchers (see, for example, [7], [8], [9], [10] and [14]). These integrals contribute significantly to the study of fracture and defects. Conservation laws may also be useful in the improvement of algorithms in numerical solutions, the establishment of global existence and uniqueness theorems, the study of stability of solutions, the investigation of phase transformation and the reduction of the order of the governing equations for different systems considered. Since there are numerous applications for conservation laws, it is of considerable interest to develop methodologies for obtaining these laws and to apply them to various systems of practical interest, including dissipative systems.

### 2. General Background

Mathematically, a conservation law of a physical system with four independent variables,  $x$ ,  $y$ ,  $z$  and  $t$ , for example, is an equation of the form

$$\text{Div } \mathbf{P} = D_x P_x + D_y P_y + D_z P_z + D_t P_t \quad (1)$$

where  $\mathbf{P} = (P_x, P_y, P_z, P_t)$  is a vector function that depends on the independent variables, the dependent variables and derivatives of the dependent variables of the system. Here  $D_i$  designates a total derivative operator. Physically, a conservation law states that the

rate of change of  $P_t$  inside any spatial domain is equal to the net flux of the "currents"  $(P_x, P_y, P_z)$  through the surface of the domain. In the special case of systems with only one independent variable, conservation laws lead to quantities which are constant on the solution manifold.

Based on variational principles, Noether's first theorem provides a method to establish conservation laws for Lagrangian systems, i.e., for systems possessing a Lagrangian function and, therefore, governed by the associated Euler-Lagrange equations. While providing a condition for existence of conservation laws, Noether's theorem by itself does not offer an explicit method for constructing divergence-free expressions. However, if one combines Noether's theorem with Lie group theory, a systematic procedure for constructing conservation laws for Lagrangian systems becomes available [2], [15] and [16].

While providing a direct procedure for obtaining conservation laws, Noether's theorem is applicable only to Lagrangian systems. In 1921, Bessel-Hagen extended Noether's theorem by introducing the concept of divergence symmetries, however, his generalization also operated only in realm of Lagrangian systems

Several isolated attempts have previously been made with the aim of deriving conservation laws for dissipation systems. Most of these start by trying to reduce the problem to a variational framework in which Noether's theorem is applicable. For example, Jiang [12] and [13] has developed conservation laws in viscoelastostatics and viscoelastodynamics by employing first Laplace's transform and then applying Noether's first theorem to the transformed equations. The conservation laws thus obtained involve convolution and when expressed in the time-domain they therefore depend on the total history of the material evolution. Later, Caviglia and Morro [3] applied the same procedure with the Bessel-Hagen extension of Noether's theorem to arrive at more general results. Soon afterwards [4], [5], they reexamined their results in view of a different procedure, called direct approach, which has been developed by Arens [1] in connection with Maxwell equations. The direct approach consists simply in integrating equation (1) subject to the constraint provided by the governing equation of the system.

In the realm of conservation laws for dissipative systems, it is worth mentioning the work of Djukic and Sutela [6] whose aim is to derive conservation laws for a finite-dimensional dynamical systems. However, these authors provide necessary but not suf-

ficient condition for the derivation of the conservation laws.

In a recent work [11], the present authors have proposed a novel approach, called the Neutral Action method, to derive, in a highly systematic fashion, conservation laws for dissipative systems governed by ordinary as well as partial differential equations. Moreover, the conditions provided for the existence of conservation laws are necessary as well as sufficient.

### 3. Statement of the problem

The problem consists in applying the Neutral Action method in order to derive conservation laws for viscoelastic bodies. In contrast with the previous attempts, emphasis will be put in constructing conservation laws in the space-time domain and the viscoelastic models considered are therefore given in terms of rate equations.

## B. SUMMARY OF THE RESULTS ACHIEVED

The "Neutral Action" method is applicable to dissipative as well as nondissipative systems.

When specialized to Lagrangian systems, it reduces to Noether's first theorem as extended by Bessel-Hagen. By applying the "Neutral Action" method we were therefore able to obtain in a unified and systematic fashion all the aforementioned results. In addition, conservation laws in the space-time domain were derived for a one-dimensional viscoelastic bar. The models considered first were: Voigt, Kelvin and Maxwell elements. Then we considered the more general case where the constitutive equation is given by

$$Q(\partial_t)\sigma = R(\partial_t)\epsilon \quad (2)$$

where  $Q$  and  $R$  are polynomials in the operator  $\partial_t$ , and  $\sigma$  and  $\epsilon$  designate the stress and strain, respectively. For the quasistatic case, we were able to show that the conservation laws are given by

$$P_x = -g'(\phi_x)\phi_t + h(\phi_x) - C\left(\frac{\phi}{\phi_x} - x\right) \quad (3)$$

$$P_t = g(\phi_x) \quad (4)$$

where  $\phi$  is defined by  $\phi = R(\partial_t)u$ ,  $u$  being the displacement field and  $g$  and  $h$  are arbitrary functions, while  $C$  is an arbitrary constant. A prime designates differentiation with respect to the arguments.

By suitable choices of  $g$  and  $h$ , different conservation laws can be obtained. One of them is the statement of energy conservation.

A generalization to two-dimensional linear isotropic viscoelastic materials has been carried out for the Voigt model.

One of the conservation laws obtained is given by

$$P_x = A[\sigma_{xx}u_{,t} + \sigma_{xy}v_{,t} + u\sigma_{xx,t} + v\sigma_{xy,t}]$$

$$P_y = A[\sigma_{yy}v_{,t} + \sigma_{xy}u_{,t} + v\sigma_{yy,t} + u\sigma_{xy,t}]$$

$$P^t = -A \{ 2W^e + (\alpha + 2\beta)(u_{,x}u_{,xt} + v_{,y}v_{,yt}) + \alpha(v_{,y}u_{,xt} + u_{,x}v_{,yt}) \\ + \beta(u_{,y}u_{,yt} + v_{,x}v_{,xt} + v_{,x}u_{,yt} + v_{,y}v_{,xt}) \}$$

where  $A$  is an arbitrary constant,  $W^e$  is the elastic strain energy and  $\alpha$ ,  $\beta$  are the coefficients of dissipation. One can verify that this conservation law is nothing but the statement of energy conservation.

C. LIST OF ALL PUBLICATIONS AND TECHNICAL REPORTS

Honein, T. and Herrmann, G., "Conservation Laws in Viscoelasticity", to appear.

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## Bibliography

- [1] Arens, R., 1983, "The conserved currents for the Maxwellian field", *Comm. Math. Phys.*, 90: 527-544.
- [2] Bluman, G. W. and S. Kumei, 1989. *Symmetries and Differential Equations*, Springer-Verlag, New York.
- [3] Caviglia, G. and A. Morro, 1988. "A General Approach to Conservation Laws in Viscoelasticity", *Acta Mechanica*, 75: 255-267.
- [4] Caviglia, G. and A. Morro, 1990, "On the Generation of Conservation Laws in Viscoelasticity", *Acta Mechanica*, 81: 91-95.
- [5] Caviglia, G. and A. Morro, 1990, "Conservation Laws in Viscoelasticity", *Quart. Appl. Math.*, 38(3): 503-516.
- [6] Djukic, D. S. and T. Sutela, 1984. "Integrating Factors and Conservation Laws for Nonconservative Dynamical Systems", *Int. J. Non-linear Mech.*, 19: 331-339.
- [7] Eshelby, J. D., 1956, "The Continuum Theory of Lattice Defects". *Solid State Physics*, 3: 79-144, F. Seitz and D. Turnbull eds., Academic Press, New York.
- [8] Fletcher, D. C., 1976, "Conservation Laws in Linear Elastodynamics", *Arch. Rational Mech. Anal.*, 60: 329-353.
- [9] Günther, W., 1962, "Über Einige Randintegrale der Elastomechanik", *Abhandlungen der Branschweigischen Wissenschaftlichen Gesellschaft*, 14: 53-72. Verlag Friedr. Vieweg, Branschweig.
- [10] Gurtin, M. E., 1979, "On a Path-Independent Integral for Thermoelasticity", *Int. J. of Fracture*, 15: R169- R170.
- [11] Honein, T. and G. Herrmann, 1991. "On Conservation Laws for dissipative systems". *Physics letters A*, 155, No. 4.5.
- [12] Jiang, Q., 1985, "On Conservation Laws in Viscoelastostatics". *Acta Mechanica*, 56: 219-227.
- [13] Jiang, Q., 1986, "Conservation Laws in Linear Viscoelastodynamics". *J. of Elasticity*, 16: 213-219.
- [14] Knowles, J. K. and E. Sternberg, 1972. "On a Class of Conservation Laws in Linearized

- and Finite Elastostatics", *Arch. Rational Mech. Anal.*, 44: 187-211.
- [15] Logan, J. D., 1977, *Invariant Variational Principles*. Academic Press, New York.
- [16] Olver, P. J., 1980, *Applications of Lie Groups to Differential Equations*, Springer-Verlag, New York.